國 立 東 華 大 學 應 用 數 學 系

專 題 演 講

一、主講人：Professor Wataru Takahashi
Tokyo Institute of Technology, Japan

講　　題：An Open Problem in Fixed Point Theory and Convex Analysis

時　　間：99年3月12日(星期五) 15:10-16:00
17:10-17:50問題討論

摘　要

Let H be a real Hilbert space with inner product $\langle \cdot,\cdot \rangle$ and norm $\|\cdot\|$ and let C be a closed convex subset of H. Let T be a mapping of C into itself. Then we denote by $F(T)$ the set of fixed points of T. A mapping $T : C \to C$ is called nonexpansive if $\|Tx - Ty\| \le \|x - y\|$ for all $x, y \in C$. A mapping $F : C \to C$ is also said to be firmly nonexpansive if $\|Fx - Fy\|^2 \le \langle x - y, Fx - Fy \rangle$ for all $x, y \in C$. In 1980, Ray (W. O. Ray, *The fixed point property and unbounded sets in Hilbert space*, Trans. Amer. Math. Soc. **258**, (1980), 531–537.) proved the following theorem.

Theorem 1. *Let H be a Hilbert space and let C be a nonempty closed convex subset of H. Then, the following are equivalent:*

(i) *Every nonexpansive mapping of C into itself has a fixed point in C;*

(ii) *C is bounded.*

We know that a nonexpansive mapping is deduced from a firmly nonexpansive mapping. Recently, Takahashi defined the following nonlinear mapping $S : C \to C$ called hybrid which is also deduced from a firmly nonexpansive mapping:

$$3\|Sx - Sy\|^2 \le \|x - y\|^2 + \|x - Sy\|^2 + \|y - Sx\|^2$$

for all $x, y \in C$. Using Ray's theorem, Takahashi proved the following theorem.

Theorem 2. *Let H be a Hilbert space and let C be a nonempty closed convex subset of H. Then, the following are equivalent:*

(i) *Every hybrid mapping of C into itself has a fixed point in C;*

(ii) *C is bounded.*

However, such theorems have not been extended to those of a Banach space. Recently, Kohsaka and Takahashi (F. Kohsaka and W. Takahashi, *Fixed point theorems for a class of nonlinear mappings related to maximal monotone operators in Banach spaces*, Arch. Math. **91** (2008), 166-177.) introduced the following

nonlinear mapping in a Banach space. Let E be a smooth, strictly convex and reflexive Banach space, let J be the duality mapping of E and let C be a nonempty closed convex subset of E. Then, a mapping $S : C \to C$ is said to be nonspreading if

$$\phi(Sx, Sy) + \phi(Sy, Sx) \le \phi(Sx, y) + \phi(Sy, x)$$

for all $x, y \in C$, where $\phi(x, y) = \|x\|^2 - 2\langle x, Jy\rangle + \|y\|^2$ for all $x, y \in E$. They proved a fixed point theorem for such mappings. In the case when E is a Hilbert space, we know that $\phi(x, y) = \|x - y\|^2$ for all $x, y \in E$. So, a nonspreading mapping S in a Hilbert space H is defined as follows:

$$2\|Sx - Sy\|^2 \le \|Sx - y\|^2 + \|Sy - x\|^2$$

for all $x, y \in C$.

In this talk, motivated by these results, we try to extend Ray's theorem to that in a Banach space by the theory of convex analysis. Then, we solve the open problem.

二、 主講人：Professor Mau- Hsiang Shih施茂祥教授

Department of Mathematics

National Taiwan Normal University

講　題：Neural Synchrony

時　間：99 年 3 月 12 日(星期五) 16:10-17:00

18:00-18:40問題討論

摘　要

We explore an evolutionary network model of pulse-coupled neurons in which the changes of evolutionary coupling strengths are based on Hebbian synaptic plasticity. A mathematical proof asserts that Hebbian synaptic plasticity is a source of causing the evolutionary network's nodal-and-coupling dynamics to tend spontaneously towards neural synchrony. These developments may lead to an explanation of nonlinear feedback effect of neural synchrony: neural synchrony allows positive feedback from which a monotonically amplifying sequence of coupling strengths and a monotonically expanding sequence of regions of neural firing states for initializing the stability process of neural synchrony arise.

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